GRIDS: Interactive Layout Design with Integer Programming
Mathematical Formulation

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1 Introduction

This document forms the supplementary material accompanying the paper submitted to CHI 2020. It provides the complete core mixed integer linear programming formulation outlined in our paper.

2 Core Mixed Integer Linear Programming Formulation

2.1 Notations and Terminologies

The origin of axes is assumed to be at the top left corner of the canvas, with Y-axis downward positive and X-axis rightwards positive. The following sets and indices are defined for the problem:

1. Size of canvas $\longrightarrow$ Width $\mathbb{W}$ and Height $\mathbb{H}$
2. Set of all elements to be placed $\longrightarrow \mathbb{E}$, where $\|\mathbb{E}\| = n$
3. Indices for element $\longrightarrow e$ and $\bar{e}$
4. Maximum prescribed width of Element $e \longrightarrow W_e^{\max}$
5. Maximum prescribed height of Element $e \longrightarrow H_e^{\max}$
6. Minimum prescribed width of Element $e \longrightarrow W_e^{\min}$
7. Minimum prescribed height of Element $e \longrightarrow H_e^{\min}$
8. Prescribed aspect ratio (width/height) of Element $e \longrightarrow \mathbb{R}_e$ This is an optional parameter. It is possible that aspect ratio is not specified for some or all element. Unspecified aspect ratios indicate that the widths and heights of such element need not be linked.

2.2 Decision variables

The decision variables governing the MILP are as follows:

$$
\begin{aligned}
L_e &:= \text{Placement of Left edge of Element } e \\
R_e &:= \text{Placement of Right edge of Element } e \\
T_e &:= \text{Placement of Top edge of Element } e \\
B_e &:= \text{Placement of Bottom edge of Element } e \\
W_e &:= \text{Actual width of Element } e \\
H_e &:= \text{Actual Height of Element } e \\
V_T^i &:= \text{Value (distance) of the TG } i \\
V_B^i &:= \text{Value (distance) of the BG } i \\
V_L^i &:= \text{Value (distance) of the LG } i \\
V_R^i &:= \text{Value (distance) of the RG } i
\end{aligned}
$$

$$
\begin{aligned}
\alpha_i^e &:= \begin{cases} 1 & \ldots \text{ if element } e \text{ belongs to TG } i \\ 0 & \ldots \text{ Otherwise} \end{cases} \\
\beta_i^e &:= \begin{cases} 1 & \ldots \text{ if element } e \text{ belongs to BG } i \\ 0 & \ldots \text{ Otherwise} \end{cases} \\
\gamma_i^e &:= \begin{cases} 1 & \ldots \text{ if element } e \text{ belongs to LG } i \\ 0 & \ldots \text{ Otherwise} \end{cases} \\
\delta_i^e &:= \begin{cases} 1 & \ldots \text{ if element } e \text{ belongs to RG } i \\ 0 & \ldots \text{ Otherwise} \end{cases} \\
\epsilon_T^i &:= \begin{cases} 1 & \ldots \text{ if TG } i \text{ is actually used} \\ 0 & \ldots \text{ if TG } i \text{ is not used} \end{cases} \\
\epsilon_R^i &:= \begin{cases} 1 & \ldots \text{ if RG } i \text{ is actually used} \\ 0 & \ldots \text{ if RG } i \text{ is not used} \end{cases} \\
\epsilon_B^i &:= \begin{cases} 1 & \ldots \text{ if BG } i \text{ is actually used} \\ 0 & \ldots \text{ if BG } i \text{ is not used} \end{cases} \\
\epsilon_L^i &:= \begin{cases} 1 & \ldots \text{ if LG } i \text{ is actually used} \\ 0 & \ldots \text{ if LG } i \text{ is not used} \end{cases}
\end{aligned}
$$

We can enforce several families of constraints based on these decision variables. Primarily, we define the element sizes and the aspect ratio.

$$0 \le L_e \le \mathbb{W} - W_e^{\min} \quad \ldots \; \forall\, e \in \mathbb{E} \tag{1}$$
$$0 \le T_e \le \mathbb{H} - H_e^{\min} \quad \ldots \; \forall\, e \in \mathbb{E} \tag{2}$$
$$W_e = R_e - L_e \quad \ldots \; \forall\, e \in \mathbb{E} \tag{3}$$
$$H_e = B_e - T_e \quad \ldots \; \forall\, e \in \mathbb{E} \tag{4}$$
$$W_e^{\min} \le W_e \le W_e^{\max} \quad \ldots \; \forall\, e \in \mathbb{E} \tag{5}$$
$$H_e^{\min} \le H_e \le H_e^{\max} \quad \ldots \; \forall\, e \in \mathbb{E} \tag{6}$$
$$H_e = \mathbb{R}_e\, W_e \quad \ldots \; \forall\, e \in \mathbb{E} \tag{7}$$

While other constraints listed above are self-explanatory, we discuss Equation (7) further. For images or pictures, the aspect ratio is strict and must be specified as an equality constraint. However, for other UI elements (such as text-fields), the aspect ratio is often an approximate guideline rather than an precise specification. In such cases, Equation (7) can be replaced by two inequality constraints that restrict the actual aspect ratio to be within a permissible interval around the specified value.

Next, we enforce every element to belong to exactly four alignment-groups (one of every type):

$$\sum_i \beta_i^e = 1 \ \ldots \ \forall e \in \mathbb{E} \tag{8}$$

$$\sum_i \alpha_i^e = 1 \ \ldots \ \forall e \in \mathbb{E} \tag{9}$$

$$\sum_i \gamma_i^e = 1 \ \ldots \ \forall e \in \mathbb{E} \tag{10}$$

$$\sum_i \delta_i^e = 1 \ \ldots \ \forall e \in \mathbb{E} \tag{11}$$

Next, we enforce the dimensions and locations of elements to match their alignment-groups:

$$L_e \ \geq V_L^i - \mathbb{W}\ (1 - \gamma_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{12}$$

$$L_e \ \leq V_L^i + \mathbb{W}\ (1 - \gamma_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{13}$$

$$R_e \ \geq V_R^i - \mathbb{W}\ (1 - \delta_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{14}$$

$$R_e \ \leq V_R^i + \mathbb{W}\ (1 - \delta_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{15}$$

$$T_e \ \geq V_T^i - \mathbb{H}\ (1 - \alpha_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{16}$$

$$T_e \ \leq V_T^i + \mathbb{H}\ (1 - \alpha_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{17}$$

$$B_e \ \geq V_B^i - \mathbb{H}\ (1 - \beta_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{18}$$

$$B_e \ \leq V_B^i + \mathbb{H}\ (1 - \beta_i^e) \ \ldots \ \forall e \in \mathbb{E}\ , \forall i \tag{19}$$

We also introduce several *interconnecting* constraints to ensure that the values of decision variables are logically correlated with each other:

$$\parallel \mathbb{E} \parallel \epsilon_B^i \ \geq \ \sum_{e \in \mathbb{E}} \beta_i^e \ \ldots \ \forall i \tag{20}$$

$$\parallel \mathbb{E} \parallel \epsilon_T^i \ \geq \ \sum_{e \in \mathbb{E}} \alpha_i^e \ \ldots \ \forall i \tag{21}$$

$$\parallel \mathbb{E} \parallel \epsilon_L^i \ \geq \ \sum_{e \in \mathbb{E}} \gamma_i^e \ \ldots \ \forall i \tag{22}$$

$$\parallel \mathbb{E} \parallel \epsilon_R^i \ \geq \ \sum_{e \in \mathbb{E}} \delta_i^e \ \ldots \ \forall i \tag{23}$$

$$\epsilon_B^i \ \leq \ \sum_{e \in \mathbb{E}} \beta_i^e \ \ldots \ \forall i \tag{24}$$

$$\epsilon_T^i \ \leq \ \sum_{e \in \mathbb{E}} \alpha_i^e \ \ldots \ \forall i \tag{25}$$

$$\epsilon_L^i \ \leq \ \sum_{e \in \mathbb{E}} \gamma_i^e \ \ldots \ \forall i \tag{26}$$

$$\epsilon_R^i \ \leq \ \sum_{e \in \mathbb{E}} \delta_i^e \ \ldots \ \forall i \tag{27}$$

The above equations ensure that the the values of ϵ are 1 if and only if one or more elements are aligned to the concerned grid line.

We also know some bounds on the minimum permissible number of alignment-groups that will be required. While the following constraints are not strictly necessary for the optimizer, they help in improving the performance of branch-and-bound tree by avoiding search spaces where integer-feasible solutions cannot exist:

$$\sum_i \epsilon_T^i + \sum_i \epsilon_L^i \geq 2 * \sqrt{\| \mathbb{E} \|} \tag{28}$$

$$\sum_i \epsilon_T^i + \sum_i \epsilon_R^i \geq 2 * \sqrt{\| \mathbb{E} \|} \tag{29}$$

$$\sum_i \epsilon_B^i + \sum_i \epsilon_L^i \geq 2 * \sqrt{\| \mathbb{E} \|} \tag{30}$$

$$\sum_i \epsilon_B^i + \sum_i \epsilon_R^i \geq 2 * \sqrt{\| \mathbb{E} \|} \tag{31}$$

Next, we consider the transition distances (Manhattan distance instead of Euclidean) between any specific pair of elements. We use the following decision variables to capture this distance:

$$DX_{e\bar{e}} := \text{Minimum horizontal distance between } e \text{and } \bar{e}$$
$$DY_{e\bar{e}} := \text{Minimum vertical distance between } e \text{and } \bar{e}$$

The distances are calculated by the following constraints:

$$DX_{e\bar{e}} \geq L_e \ - L_{\bar{e}} \ - W_{\bar{e}} \quad \ldots \ \forall e, \bar{e} \in \mathbb{E} \tag{32}$$

$$DX_{e\bar{e}} \geq L_{\bar{e}} \ - L_e \ - W_e \quad \ldots \ \forall e, \bar{e} \in \mathbb{E} \tag{33}$$

$$DY_{e\bar{e}} \geq T_e \ - T_{\bar{e}} \ - H_{\bar{e}} \quad \ldots \ \forall e, \bar{e} \in \mathbb{E} \tag{34}$$

$$DY_{e\bar{e}} \geq T_{\bar{e}} \ - T_e \ - H_e \quad \ldots \ \forall e, \bar{e} \in \mathbb{E} \tag{35}$$

The decision variables listed above are sufficient to capture the alignment aspect of objective functions as discussed earlier. Minimization of summation of ϵ_B, ϵ_T, ϵ_R, ϵ_L will ensure a well-aligned layout of all elements. The distance variables DX, DY allow a direct minimization of the weighted transition distances. In conjunction with alignment-groups, this ensures that closely inter-related elements be placed aligned and in close proximity.

Lastly, we look at our objective of ensuring an overall rectangular outline for the external hull. We do not introduce any new decision variables for this; instead we use our existing alignment groups with the additional intention.

Our intuition here is that the smallest rectangular outline (SRO) is exactly defined by the four extreme alignment groups. For example, the smallest possible value of V_L represents the left-most gridline and this matches the left edge of the SRO. Similarly, the smallest value of V_T in conjunction with the largest values of V_B and V_R define the SRO completely. For notation purpose, we restrict that the extreme alignment groups be defined by index 0 each (i.e. V_L^0, V_R^0, V_T^0, V_B^0). Then we can specify that $\epsilon_L^0 = \epsilon_R^0 = \epsilon_T^0 = \epsilon_B^0 = 1$. We enforce these designated gridlines to indeed be used at extremities by using the following constraints:

$$V_L^0 \leq V_L^i \ \ldots \forall \ i \neq 0 \tag{36}$$

$$V_T^0 \leq V_T^i \ \ldots \forall \ i \neq 0 \tag{37}$$

$$V_R^0 \geq V_R^i \ \ldots \forall \ i \neq 0 \tag{38}$$

$$V_B^0 \geq V_B^i \ \ldots \forall \ i \neq 0 \tag{39}$$

$$\tag{40}$$

In light of this notation for the extreme gridlines, we now consider the group-membership variables γ_0, δ_0, α_0, β_0. If any element $e$has any one of its edges aligned with the SRO, it must have the corresponding group-membership variable to be 1. On the contrary, 0 value for all these extreme gridline variables indicates an element at the interior of the layout. This fact can be used (as explained in Section 4 of the paper).

For example, the first option of rewarding any adherence to rectangular extremities is enforced by maximizing the summation of γ_0, δ_0, α_0 and β_0 over all elements. After this optima $\mathbb{R}_{min}$ is calculated in Step 5 of Algorithm 1, it is enforced as a constraint from step 6 onwards to ensure overall rectangular alignment. There are potential pifalls to this approach; in larger and complex instances, the optimizer prefers solutions where smaller sized elements are at the periphery and larger sized elements are in the interior of the layout. For such cases, the other options specified for enforcing rectangularity can be similarly enforced.